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ORIGINAL CONTRIBUTION

Translation of two coaxial, nonhomogeneously structured flocs normal to a plate

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Abstract The translation of two identical, coaxial, nonhomogeneously structured flocs in a Newtonian fluid normal to a plate is analyzed theoretically for the case where the Reynolds number ranges from 0.1 to 40. The geometry considered is capable of simulating the behavior of a floc near a boundary when other flocs may present. The description of the structure of a floc is based on a two-layer model where the permeability of its inner layer can be different from that of its outer layer. We show that the influence of the plate on the flow field near the near floc is more significant than that near the far floc, but the behavior of the corrected drag coefficient of the far floc is more complicated than that of the near floc. In general, the presence of the plate has the effect of raising the drag on a floc. The more nonhomogeneous the floc structure is, the more appreciable the deviation of the corrected drag coefficient–Reynolds number curve from the corresponding Stokes' law-like relation. An empirical relation, which correlates the corrected drag coefficient with the Reynolds number, the separation distance between two flocs, the floc–floc distance, and the ratio permeability of outer layer to permeability of inner layer, is proposed.

Keywords Sedimentation · Flocs · Nonhomogeneously structured · Boundary effect · Two flocs normal to plate · Drag

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Introduction

The translation of a particle through a liquid medium is a phenomenon common to many operations and processes in chemical, biomedical, and environmental engineering and science. Practical applications including, for example, sedimentation, flotation, electrophoresis, aggregation, flocculation, and blood flow, all involve this phenomenon. One of the key steps in modeling that phenomenon is to evaluate the drag acting on a particle. In practice, two factors often need be considered, namely, the presence of a boundary and nearby particles. The former one can be significant if the available working space is limited or the particle is close to a boundary such as in the settling of flocs formed in wastewater treatment process near the settling tank. The latter factor should not be ignored if the concentration of particles is appreciable so that the interaction between neighboring particles is important. In general, those two factors will influence the flow field near the target particle in that the drag acting on its surface is raised. In a study of the translation of a rigid sphere normal to a rigid plate in an infinite viscous fluid under creeping flow conditions, Happel and Brenner [1] found that the drag on the sphere increases with decreasing sphere-plate distance. Based on a singular perturbation approach, Cox and Brenner [2] evaluated the hydrodynamic force acting on a rigid sphere moving perpendicular to a rigid plane taking the inertial effect into account for the case where the Reynolds number is small. Stimson and Jeffery [3] studied the hydrodynamic interactions between two identical, rigid, coaxial spheres moving along the line of their centers in an unbounded viscous fluid. They showed that under creeping flow conditions, the drag acting on each sphere is the same, and the shorter the distance between them, the smaller the drag. The results of Stimson and Jeffery [3] were justified experimentally by Happel and Pfeffer [4], in which the falling of two spheres in a viscous liquid was conducted. Daugan et al. [5, 6] investigated experimentally the settling of two or three identical rigid spheres along their line of centers in a shear-thinning fluid for the case where the Reynolds number is smaller than 10^{-2}. Based on the results of two spheres, they suggested that there exists a critical initial separation distance under which the spheres form a chained doublet. Hsu and Yeh [7] calculated the drag acting on two rigid, coaxial spheres moving along the axis of a cylindrical tube filled with a shear-thinning Carreau fluid. They showed that the boundary effect for the case of the Carreau fluid is more significant than that for the case of the corresponding Newtonian fluid. Several attempts were made on the measurements of the forces acting on suspended particles at intermediate to high Reynolds number [8–14].

Compared with the available results for the drag acting on rigid particles, those acting on porous particles are much limited. Theoretically, this is because solving the governing equations for the case of porous particles is more difficult than solving those for the case of rigid particles because the domain inside a particle needs be considered in the former. Experimentally, the procedure for measuring the drag acting on a porous particle is much more delicate than that on a rigid particle because the structure of a particle can play a role in the former factor. Floc formed in water and wastewater treatment, for example, is highly porous and fragile [15, 16], and measuring the drag acting on such particles accurately is challenging. Several attempts were made on the measuring [17–20] and modeling [21–30] the drag on porous particles.

To investigate simultaneously the boundary effect and the influence of neighboring flocs on the drag acting on a floc as it translates through a liquid medium, we consider the case of two identical, coaxial, nonhomogeneously structured spheres flocs moving normal to a plate. The structure of a floc is simulated by a two-layer model [27–33]. Through adjusting the relative permeability of the inner and that of the outer layer of the floc, various possible structures [15, 16, 20, 34, 35] can be described. The influences of the key parameters of the system under consideration, including the separation distance between two flocs, the distance between flocs and plate, the Reynolds number, and the physical properties of flocs, on the drag acting on the flocs are investigated.

Theory

Let us consider the translation of two identical, coaxial, nonhomogeneously structured spherical flocs in a Newtonian fluid normal to a plate illustrated in Fig. 1. Let h, d, and S be the distance between the center of the near floc and the plate, the diameter of a floc, and the center-to-center distance between two flocs, respectively. A floc comprises of an inner layer of radius r_i and permeability k_i and an outer layer of radius r_o (=d/2) and permeability k_o. For convenience, the flocs are fixed in the space and the surrounding fluid flows with a relative velocity V.

Suppose that the fluid is incompressible with constant physical properties and Darcy–Brinkman model [21] is applicable, then the flow field at steady state can be described by [36]:

$$\mathbf{u}_f \cdot \nabla \mathbf{u}_f = -\nabla P + \frac{2}{Re}\nabla^2 \mathbf{u}_f, \text{ outside flocs} \quad (1)$$

$$\nabla \cdot \mathbf{u}_f = 0, \text{ outside flocs} \quad (2)$$

$$\mathbf{u}_j + \frac{Re}{2\beta_j^2}\nabla P = \nabla^2 \mathbf{u}_j, j = o, i \quad (3)$$

$$\nabla \cdot \mathbf{u}_f = 0, j = o, i \quad (4)$$

Here, $Re = 2\rho r_o V_z / \mu_f$ is the Reynolds number with ρ, μ_f, and V_z that are the density and the viscosity of fluid and the z-component of V, respectively. $P = (p + \rho g Z)/\rho V_z^2$ is the scaled pressure with p, g, and Z that are the pressure, the gravitational acceleration, and the z-coordinate, respectively. ∇ is the dimensionless gradient operator scaled by

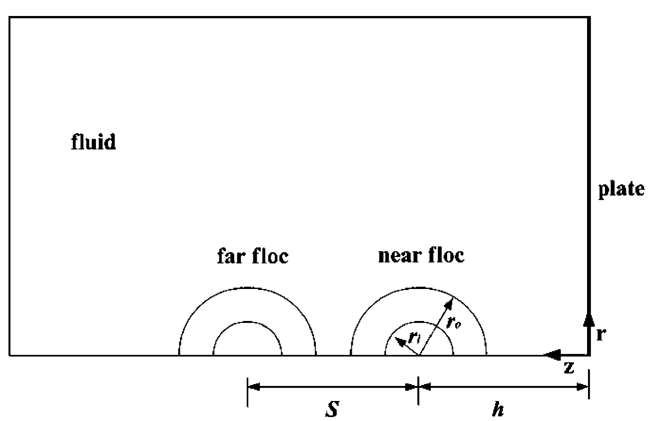

Fig. 1 Translation of two identical, coaxial, nonhomogeneously structured, spherical flocs normal to a rigid plate: h, r_i, r_o, and S are the distance between the center of the near floc and the plate, the radii of the inner and the outer layers of a floc, and the center-to-center distance between two flocs, respectively. V is the relative velocity of the flocs

$(1/r_o)$, and u_f is the dimensionless velocity scaled by V. j is a region index, $j=o$ and i represent, the outer and the inner layer of a floc, respectively, $\beta_j = r_o/\sqrt{k_j}$ is the scaled particle radius, and u_j is the scaled fluid velocity in region j scaled by V.

The following boundary conditions are assumed:

$$u_z = 1 \text{ as } r \to \infty \tag{5}$$

$$u_z = 1 \text{ at } z = 0 \tag{6}$$

$$\mathbf{u}_f = \mathbf{u}_o \text{ and } \nabla u_f = \nabla u_o \text{ at } r = r_o \tag{7}$$

$$\mathbf{u}_o = \mathbf{u}_i \text{ and } \nabla u_o = \nabla u_i \text{ at } r = r_i \tag{8}$$

$$\frac{\partial u_f}{\partial r} = \frac{\partial u_o}{\partial r} = \frac{\partial u_i}{\partial r} = 0 \text{ at } r = 0 \tag{9}$$

where u_z is the magnitude of the dimensionless fluid velocity in the z-direction. For simplicity, we assume that the viscosity of the fluid inside a floc is the same as that outside it.

For the present case, the hydrodynamic drag acting on a floc, F, can be expressed as [21]

$$F = \left(\frac{1}{2}\rho V^2\right)\left(\pi r_o^2\right) C_D \Omega \tag{10}$$

where C_D is the drag coefficient and Ω is a correction factor, $0 < \Omega \leq 1$, with equality applies to a rigid sphere. Similar to the case of Stokes' law [36], we define [31]:

$$C_D \Omega = \frac{A'}{Re} \tag{11}$$

where A' depends on the nature of a floc, the separation distance between two flocs, and that between flocs and plate.

The present problem is solved numerically by FIDAP 7.6, a commercial software based on finite element scheme. Numerical simulations are conducted to examine the variation of the drag acting on a floc under various conditions. Double precision is used and grid independence is checked throughout the simulation. Typically, using 12,850, 30, and 35 for the numbers of elements in the liquid phase, the inner layer of a floc, and the outer layer of a floc, respectively, is sufficient. To justify the applicability of the software adopted, it is used to evaluate the drag acting on two identical rigid spheres translating in an unbound Newtonian fluid normal to a plane and that on a rigid sphere translating in a Newtonian fluid normal to a plane, where analytical results are available [1]. Figure 2 shows the analytical results of Happel and Brenner [1] and the corresponding numerical results based on the software used. As seen in this figure, the performance of the numerical approach adopted is satisfactory.

However, the numerical approach adopted has limitations on the lower limit of S and h_n/r_o. Let S/d and h_n/r_o be the scaled distance between two flocs and that between the near floc and the plate, respectively, then, if S/d=1, two

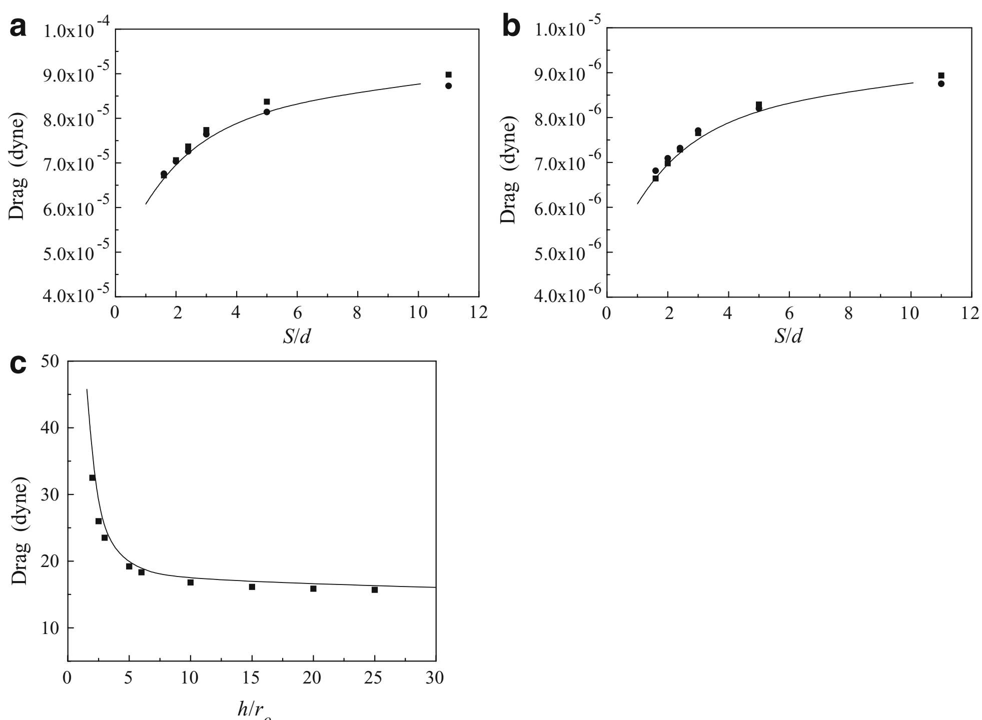

Fig. 2 Variation of the drag on two rigid spheres moving coaxially in an unbounded fluid as a function of S/d (**a**, **b**) and that on a single sphere moving normal to a plate as a function of h/r_o (**c**). *Solid line* Analytical result of Happel and Brenner [1]; *discrete symbols* present result; *filled square* near or single sphere; *filled circle* far sphere. Re=0.1 in **a** and **c**, and Re=0.01 in **b**

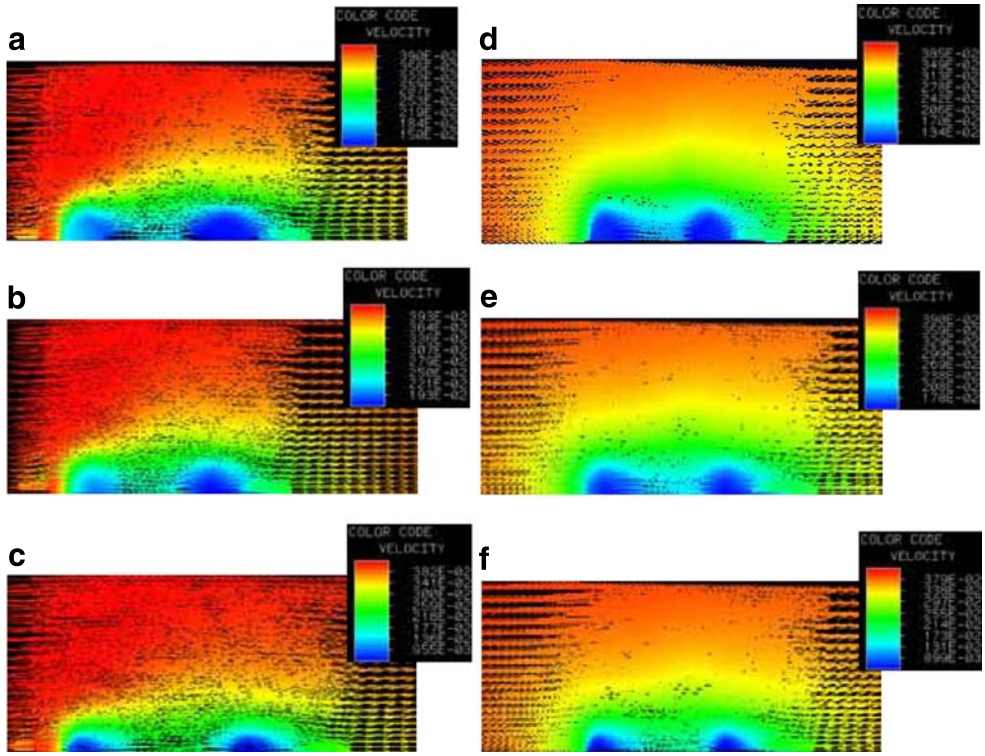

Fig. 3 Flow fields at various combinations of k_o/k_i and h_n/r_o at S/d=2, $\overline{\beta} = 2$, and Re=0.1. h_n/r_o=2 in **a**–**c**, and h_n/r_o=10 in **d**–**f**. k_o/k_i=0.1 in **a** and **d**, k_o/k_i= 1 in **b** and **e**, and k_o/k_i=10 in **c** and **f**

flocs will touch each other, and if h_n/r_o=1, the near floc will touch the plate. In our case, appreciable scattering is observed for the numerical results obtained for $1 < S/d < 2$ and/or $1 < h_n/r_o < 2$, and they are unreliable. Therefore, the lower limit of two is used for both S/d and h_n/r_o in the numerical simulations. On the other hand, because the interaction between two entities in a flow field is usually unimportant if the separation distance between them is on the order of ten times the linear size of an entity, therefore, the upper limit of ten is assumed for both S/d and h_n/r_o, and this will be justified later.

Results and discussion

The following values are assumed in subsequent discussions: r_o=0.12 cm, r_i=0.06 cm, ρ=1 g/cm^3, and μ=0.01 poise. For convenience, we define the volume-average radius $\overline{\beta}$ and the volume-average permeability $\overline{k}$ of a floc as

$$\overline{\beta} = r_o \Big/ \sqrt{\overline{k}} \tag{12}$$

$$\overline{k} = \sum_{j=1}^{2} V_j k_j \Big/ \sum_{j=1}^{2} V_j \tag{13}$$

where V_j is the volume of the jth layer of the floc. In addition, subscripts n and f refer to the properties of the near and the far flocs, respectively.

Flow field

Figures 3, 4, 5, and 6 show the velocity fields at various combinations of k_o/k_i, h_n/r_o, and S/d at two levels of Re for the case where both the radius and the volume-average permeability of both particles are fixed. As seen in Fig. 3, if Re is small, the flow field in the upstream of both flocs is symmetric to that in their downstream, implying that the convective motion of fluid is unimportant in this case. If the flocs are away from the plate, the flow field near the near floc is essentially the same as that near the far floc. However, if the flocs are near the plate, the flow field near the near floc becomes different from that near the far floc. Figure 4 reveals that if Re is sufficiently large, the convective term in the Navier–Stokes equation becomes significant, and the flow field in the upstream region of both flocs is no longer symmetric to that in their downstream region. The qualitative behaviors of the flow fields in Figs. 5 and 6 are similar to those in Figs. 3 and 4. Roughly, if S/d exceeds approximately 10, the interaction between two flocs becomes negligible.

Influence of boundary

Figure 7 reveals that for a fixed volume-average permeability $\overline{\beta}$, the $C_D\Omega$ of a homogeneously structured floc is smaller than that of a nonhomogeneously structured floc. This is true whether the inner layer of the floc is more permeable to its outer layer or not. Because $C_D\Omega$ increases

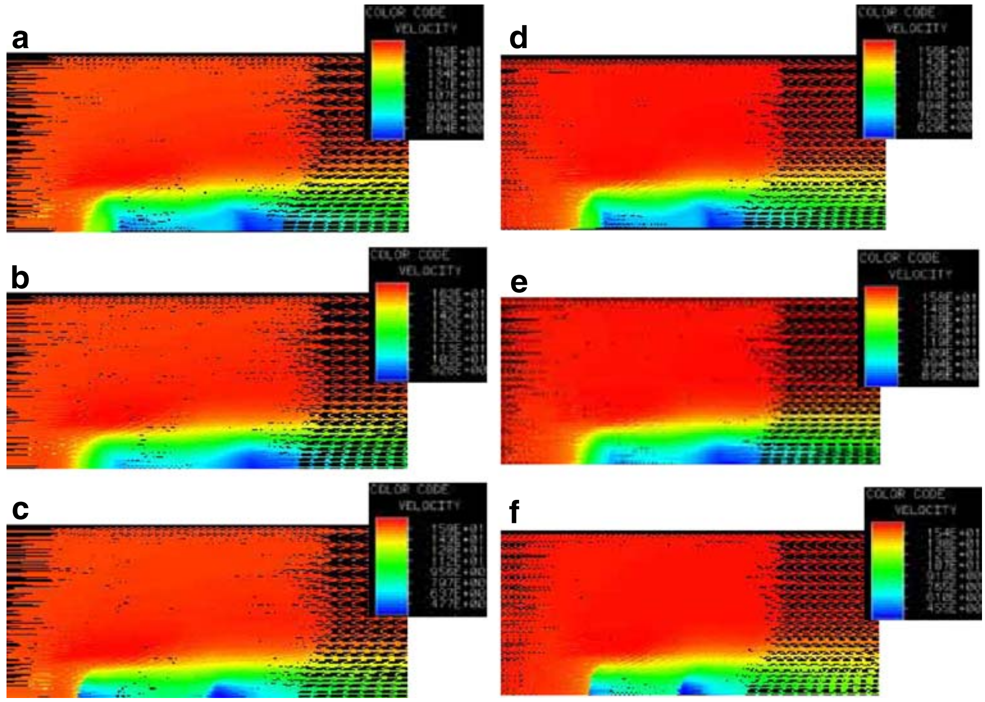

Fig. 4 Flow fields at various combinations of k_o/k_i and h_n/r_o at S/d=2, $\overline{\beta}=2$, and Re=40. h_n/r_o= 2 in **a**–**c**, and h_n/r_o=10 in **d**–**f**. k_o/k_i=0.1 in **a** and **d**, k_o/k_i=1 in **b** and **e**, and k_o/k_i=10 in **c** and **f**

with decreasing h_n/r_o, the presence of the plate has the effect of retarding the translation of the flocs. As expected, the near floc is affected more significantly. If Re is small and h_n/r_o is large, both the interaction between two flocs and the boundary effect are unimportant. In this case, $(C_D\Omega)_n$ is close to $(C_D\Omega)_f$, as is seen in Fig. 7a,b. On the other hand, if Re is sufficiently large, even if h_n/r_o is large, the interaction between two flocs is appreciable. In this case, the presence of the wakes in the rear region of the near floc has the effect of reducing the drag on the far floc, as illustrated in Fig. 7c,d. Figure 7a,b indicates that if flocs are close to the plate, even Re is small, the drags on them are different with $(C_D\Omega)_n>(C_D\Omega)_f$. A comparison between Fig. 7 panel a and panel c, or Fig. 7 panel b and panel d, suggests that increasing Re has the effect of reducing the boundary effect. In general, if S/d exceeds approximately 10, the influence of neighboring floc can be neglected.

Table 1 summarizes the boundary effect on the corrected drag coefficients of two flocs, measured by the ratio $[C_D\Omega(h_n/r_o=2)/C_D\Omega(h_n/r_o=10)]$ at various conditions, where the denominator estimates the $C_D\Omega$ when the boundary effect is absent. This table reveals that the boundary effect on the near floc declines with increasing Re, increasing S/d, and decreasing $\overline{\beta}$. Note that even if the volume-average permeability of a floc is fixed, the boundary effect still depends upon its structure, which is consistent with the observation of Hsu and Hsieh [30]. If $\overline{\beta}$ is fixed, the influence of the plate on a homogeneously structured floc is less significant than that on a nonhomogeneously structured floc; the more nonhomogeneous the floc structure, the more important the influence. As seen in Table 1, the boundary effect on the far floc is less serious than that on the near floc. Because the flow field near the far floc is affected not only by the plate but also by the near floc, the corresponding trend of the boundary effect for the far floc is not that regular as that for the near floc. In general, the smaller the Re, the more significant the boundary effect is. However, several exceptions are observed. For example, the boundary effect at S/d=10, $\overline{\beta}$= 2, and Re=0.1 is less significant than that at S/d=10, $\overline{\beta}$= 2, and Re=40. This is because at S/d=10 and h_n/r_o=10, the distance between the far floc and the plate is far, the boundary effect can be neglected, but at h_n/r_o=2, the far floc is still affected by the plate, as seen in Figs. 5 and 6. At Re=0.1, the fluid flow is slow, the flow field near the far floc is not influenced by the near floc, as illustrated in Fig. 5a–c. However, at Re=40, the flow field near the far floc is influenced both by the near floc and by the plate, leading to a greater apparent boundary effect, as seen in Fig. 6a–c. This phenomenon is pronounced if $\overline{\beta}$ is large (the volume-average permeability of a floc is small).

Influence of Reynolds number

As stated in Eq. 11, for a Newtonian fluid in the creeping flow regime, $\ln(C_D)$ is linearly proportional to $\ln(Re)$, the Stokes' law. In our case, if Re is small, $\ln(C_D\Omega)$ and $\ln(Re)$ are linearly correlated, but a positive deviation from the linear relation occurs if Re is large. Table 2 summarizes the percentage deviations of $C_D\Omega$ from a Stokes' law-like relation, defined by $\mathrm{PD}=\left[(C_D\Omega-C_D\Omega')/C_D\Omega'\right]\times 100\%$,

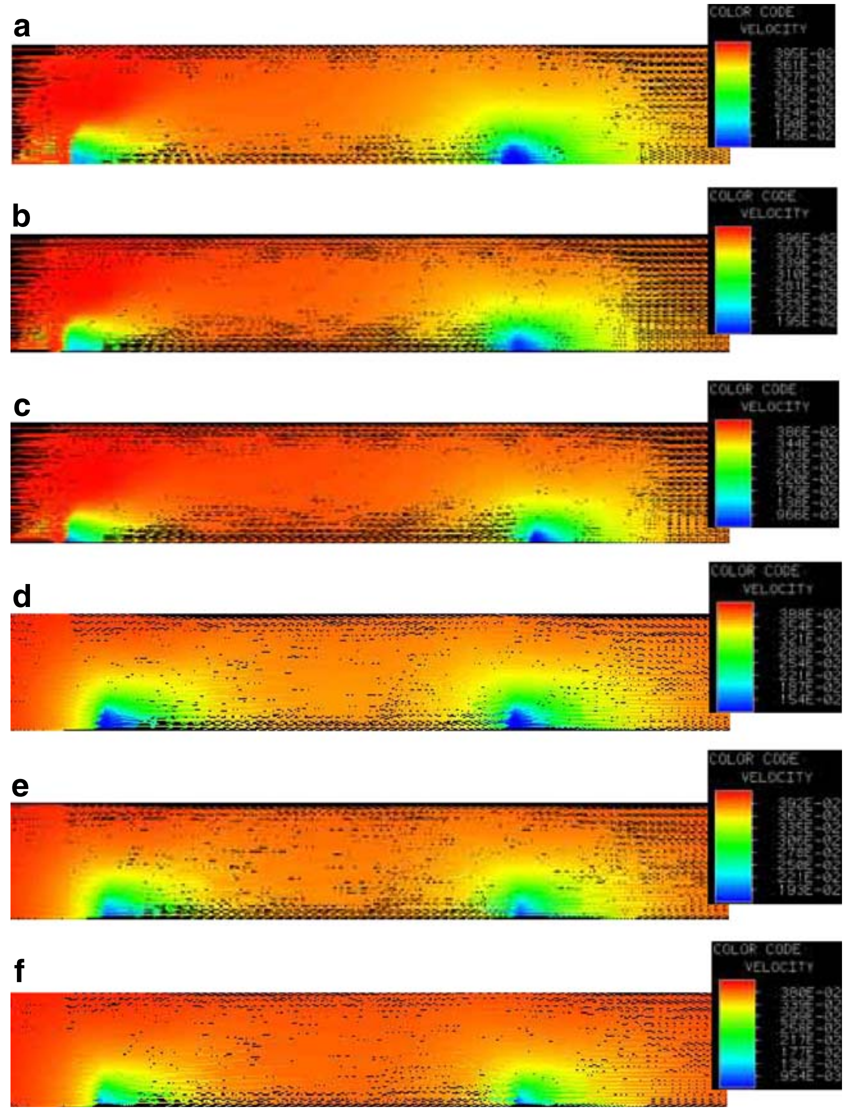

Fig. 5 Flow fields at various combinations of k_o/k_i and h_n/r_o at S/d=10, $\overline{\beta} = 2$, and Re=0.1. h_n/r_o=2 in **a**–**c**, and h_n/r_o=10 in **d**–**f**. k_o/k_i=0.1 in **a** and **d**, k_o/k_i= 1 in **b** and **e**, and k_o/k_i=10 in **c** and **f**

where $(C_D\Omega)'$ is the corrected drag coefficient based on the Stokes' law-like relation, for various combinations of S/d, h_n/r_o, and k_o/k_i at Re=40. This table reveals that the interaction between two flocs has the effect of enhancing the convective flow, leading to a larger PD. On the other hand, the presence of the plate has the effect of depressing the convective flow near a floc, yielding a smaller PD. The PD of the near floc increases with decreasing S/d or increasing h_n/r_o. The behavior of the PD of the far floc is not that regular as that of the near floc because the flow field near the far floc is affected not only by the plate but also by the flow field behind the near floc. Note that if the separation distance between two flocs is sufficiently small (S/d=2), the general trend of the PD of the far floc becomes the same as that of the near floc, both decrease with increasing significance of the boundary effect, that is, decreasing h_n/r_o. However, if the separation distance between two flocs is sufficiently large (S/d=10), although the PD of the near floc declines with decreasing h_n/r_o, that of the far floc increases with decreasing h_n/r_o. If h_n/r_o is large, the PD of the far floc is smaller than that of the near floc, but the reverse trend is observed if h_n/r_o is small. This is because if the boundary effect is insignificant, the $C_D\Omega$ of the far floc is influenced mainly by the flow field in the downstream of the near floc, and this effect is pronounced as S/d declines. Although the PD of both flocs decreases with decreasing h_n/r_o, the degree of decrease of the far floc is not that large compared with that of the near floc because the influence of the plate on the far floc is smaller than that on the near floc. Note that at S/d=2 and h_n/r_o=2, the PD of the near floc is about the same as that of the far floc. For both the near and the far flocs, the more nonhomogeneous the floc structure, the larger the PD.

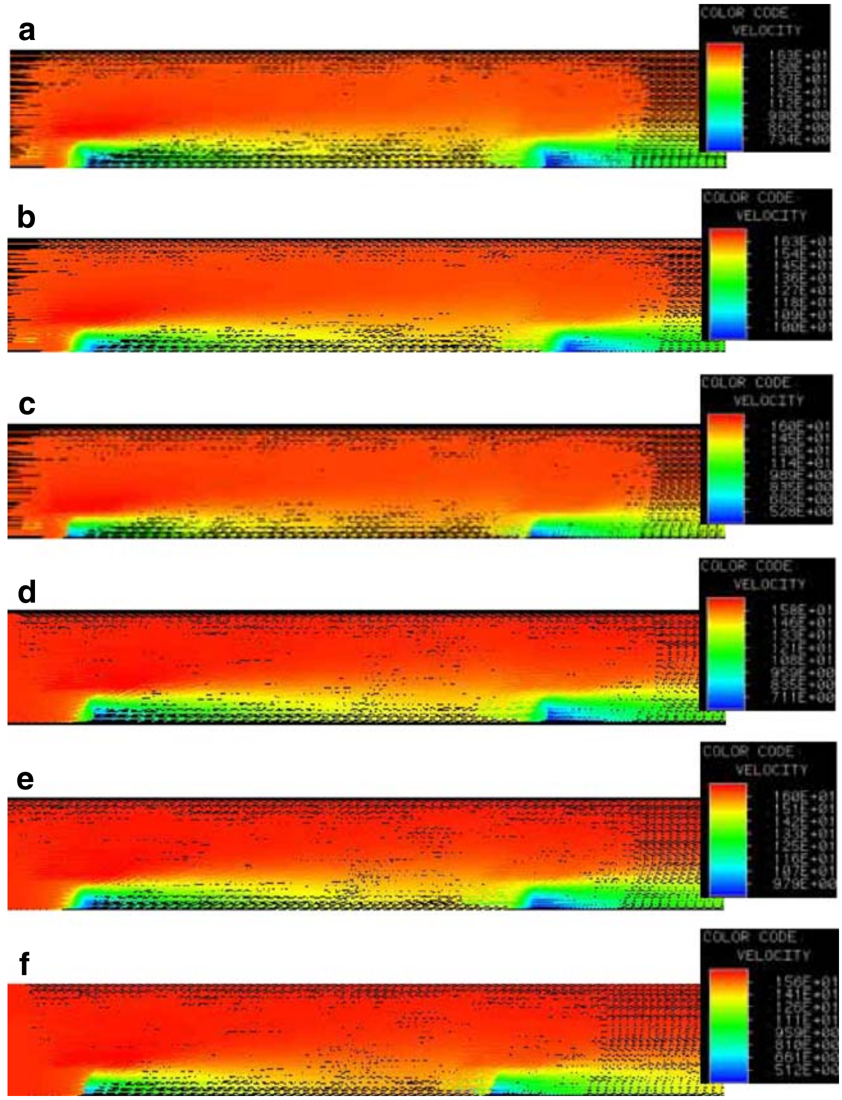

Fig. 6 Flow fields at various combinations of k_o/k_i and h_n/r_o at S/d=10, $\overline{\beta}=2$, and Re=40. h_n/r_o=2 in **a**–c, and h_n/r_o=10 in **d**–**f**. k_o/k_i=0.1 in **a** and **d**, k_o/k_i= 1 in **b** and **e**, and k_o/k_i=10 in **c** and **f**

Influence of floc structure

Figures 8 and 9 show the variations of $C_D\Omega$ of each floc as a function of r_i/r_o at various combinations of k_o/k_i, Re, and h_n/r_o. Note that curve 3 in these figures represents the results for a homogeneously structured floc. Equation 12 suggests that for fixed $\overline{\beta}$ if k_o/k_i<1, both the permeability of the outer layer and that of the inner layer of a floc decrease with increasing r_i/r_o, yielding a larger $(C_D\Omega)_n$ and $(C_D\Omega)_f$, as illustrated by curves 1 and 2 of Figs. 8 and 9. On the other hand, if k_o/k_i>1, both the permeability of the outer layer and that of the inner layer of a floc increase with increasing r_i/r_o, leading to a smaller $(C_D\Omega)_n$ and $(C_D\Omega)_f$, as seen in curves 4 and 5 of Figs. 8 and 9. Similar behavior was also observed in the translation of two particles parallel to their line of centers in an unbounded viscous fluid [28], and the translation of two porous spheres along the axis of a cylinder [29]. The dependence of $C_D\Omega$ on r_i/r_o for the case where k_o/k_i<1 is more sensitive than that for the case where k_o/k_i>1. Table 3 summarizes the influence of r_i/r_o on the corrected drag coefficient $C_D\Omega$ of each floc, measured by $\{[C_D\,\Omega\,(r_i/r_o=0.7)-C_D\Omega(r_i/r_o=0.25)]/[C_D\Omega(r_i/r_o=0.25)]\}\times 100\%$; the larger the absolute value of this ratio, the more important the effect. The general trend of the influence of r_{io} on the near floc is the same as that on the far floc: the more nonhomogeneous the floc structure and/or the larger the Re, the more significant that influence is. The degree of increase in $C_D\Omega$ as r_{io} increases is enhanced by the presence of the plate, implying that $(C_D\Omega)_n$ is more sensitive to the variation of r_{io} than $(C_D\Omega)_f$.

Correlation for $C_D\Omega$

For the case where $\overline{\beta}=2$ and r_i/r_o=0.5, the results of the corrected drag coefficient $C_D\Omega$ for the near and the far flocs over the ranges of the parameters considered in the

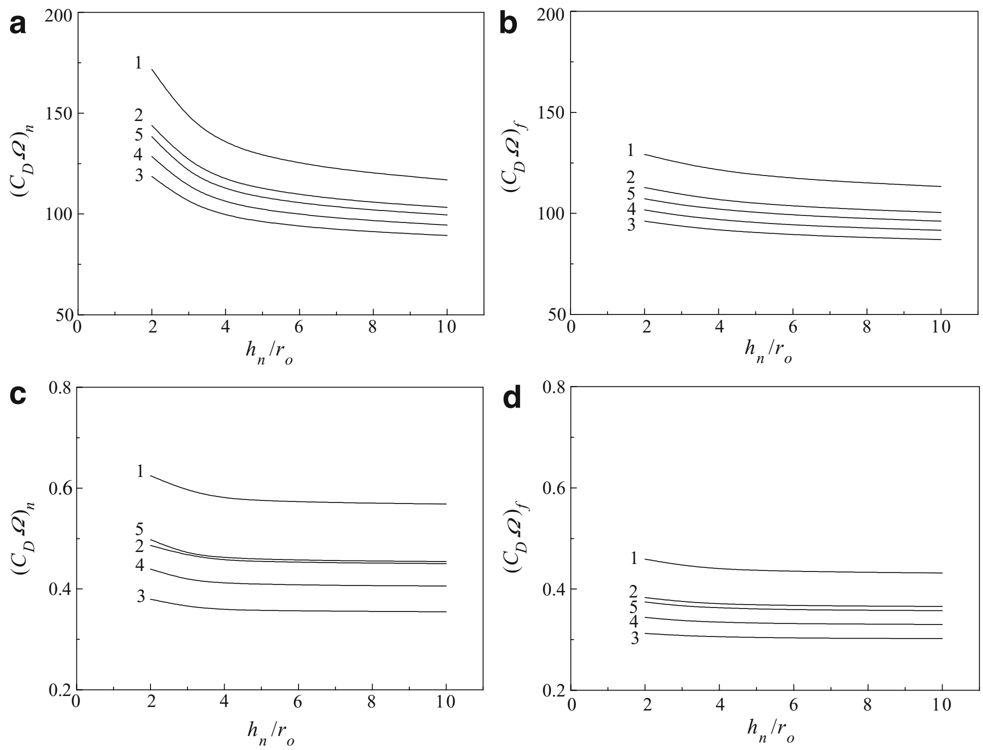

Fig. 7 Variations of $(C_D\Omega)_n$ (**a**, **c**) and $(C_D\Omega)_f$ (**b**, **d**), as a function of h_n/r_o for various combinations of k_o/k_i and *Re* at $\overline{\beta}=2$ and *S*/*d*=2. *Curve 1* k_o/k_i=0.1, *2* k_o/k_i=0.2, *3* k_o/k_i=1, *4* k_o/k_i=5, *5* k_o/k_i=10. *Re*=0.1 in **a** and **b**, *Re*=40 in **c** and **d**

numerical simulation can be summarized for $2 \leq S/d$ and $2 \leq h_n/r_0$ as

$$C_D\Omega = \frac{1}{Re}\left[\frac{8.34 + 0.075(S/d) - 0.004(S/d)^2}{1-(r_o/h)+0.9(r_o/h)^2}\right]\Phi, Re\leqq 1 \tag{14}$$

$$C_D\Omega = \left\{\sqrt{\frac{1}{Re}\left[\frac{8.34 + 0.075(S/d) - 0.004(S/d)^2}{1-(r_o/h)+0.9(r_o/h)^2}\right]} + 0.08\right\}^2\Phi, \quad 1 < Re\leqq 40 \tag{15}$$

where

$$\Phi = A + B\times(k_o/k_i) + C\times D^{(k_o/k_i)} \tag{16}$$

$$\mathrm{A} = 0.99 \tag{17}$$

$$B = \frac{0.0106 - 2.23\times10^{-5}(S/d)}{0.9-1.2(r_o/h)+1.1(r_o/h)^2} \tag{18}$$

$$C = \frac{0.52 + 0.03(S/d) - 0.002(S/d)^2}{1-1.3(r_o/h)+1.1(r_o/h)^2} \tag{19}$$

$$D = \frac{8.94\times10^{-4} - 1.71\times10^{-5}(S/d) + 1.03\times10^{-6}(S/d)^2}{0.7-0.4(r_o/h)+2.5(r_o/h)^2}. \tag{20}$$

Through replacing *h* by h_n and h_f, respectively, Eqs. 14–20 are applicable to both the near and the far flocs. For example, if h_n/r_o=2 and *S*/*d*=2, then use r_o/h=1/2 in Eqs. 14–20 for the near floc and r_o/h=1/6 for the far floc. The performance of Eqs. 14–20 is illustrated in Fig. 10; the smaller the *Re*, the better the performance. Two hundred forty data points are used to obtain Eq. 14; the average and the maximum deviations of $C_D\Omega$ are 2.71% and 8.78%, respectively. In general, the deviation is larger at the following conditions: larger *Re*, larger *S*/*d*, and the more nonhomogeneous the floc structure. In addition, the deviation for the far floc is larger than that for the near floc. The average and the maximum deviations of $C_D\Omega$ based on Eq. 15 are 7.44% and 26.21%, respectively. In general, the deviation is larger for a larger *Re* and a more nonhomogeneously structured floc.

Conclusions

The influence of boundary and the presence of nearby flocs on the drag acting on a floc is investigated by considering the translation of two identical, coaxial, spherical flocs normal to a plane in a Newtonian fluid at a small to

Table 1 Variation of the boundary effect, measured by $[C_D\Omega(h_n/r_o = 2)/C_D\Omega(h_n/r_o = 10)]$, at various combinations of (*S*/*d*), $\overline{\beta}$, *Re*, and (k_o/k_i), where the denominator estimates roughly the $(C_D\Omega)$ when the boundary effect is absent

S/*d*	$\overline{\beta}$	*Re*	k_o/k_i	$[C_D\Omega(h_n/r_o = 2)/C_D\Omega(h_n/r_o = 10)]$	
				Near floc	Far floc
2	2	0.1	0.1	1.469	1.140
			0.2	1.394	1.124
			1	1.328	1.106
			5	1.360	1.110
			10	1.389	1.116
		40	0.1	1.099	1.063
			0.2	1.081	1.048
			1	1.069	1.035
			5	1.083	1.042
			10	1.096	1.048
	1	0.1	0.1	1.200	1.070
			0.2	1.156	1.055
			1	1.101	1.042
			5	1.153	1.049
			10	1.188	1.057
		40	0.1	1.045	1.015
			0.2	1.040	1.010
			1	1.038	1.006
			5	1.045	1.008
			10	1.053	1.012
10	2	0.1	0.1	1.367	1.038
			0.2	1.308	1.037
			1	1.257	1.037
			5	1.282	1.041
			10	1.305	1.043
		40	0.1	1.083	1.067
			0.2	1.067	1.059
			1	1.057	1.054
			5	1.071	1.060
			10	1.082	1.065
	1	0.1	0.1	1.157	1.033
			0.2	1.124	1.032
			1	1.101	1.033
			5	1.124	1.038
			10	1.149	1.041
		40	0.1	1.037	1.043
			0.2	1.032	1.040
			1	1.030	1.039
			5	1.038	1.043
			10	1.045	1.047

medium large Reynolds number. The distribution of the permeability inside a floc is nonhomogeneous, and a two-layer model is adopted to simulate its structure. The governing equations for the flow field are solved numerically and the result used to evaluate the drag acting on a floc. The results of numerical simulation can be summarized as following: (a) If Reynolds number is smaller than approximately 0.1, the flow field in the upstream of a floc is symmetric to that in its downstream. In this case, if flocs are away from the plate, the flow field near the near floc is the same as that near the far floc. However, if the flocs are near the plate, they become different. If Reynolds number exceeds approximately 10, the flow field in the upstream region of a floc is no longer symmetric to that in its downstream region. The interaction between two flocs becomes negligible if the ratio center-to-center distance between two flocs to diameter of floc exceeds approximately 10. (b) If the volume-average permeability of a floc is fixed, the corrected drag coefficient of a homogeneously structured floc is smaller than that of the corresponding nonhomogeneously structured floc. (c) The boundary effect on the near floc decreases with increasing Reynolds

Table 2 Percentage deviations of $(C_D\Omega)$ from a Stokes' law-like relation at *Re*=40, defined by $\mathrm{PD} = [(C_D\Omega - C_D\Omega')/C_D\Omega'] \times 100\%$, where $(C_D\Omega)'$ is the corrected drag coefficient based on the Stokes' law-like relation, for various combinations of (*S*/*d*), (h_n/r_o), and (k_o/k_i) at $\overline{\beta} = 2$

S/*d*	h_n/r_o	k_o/k_i	PD (%)	
			Near floc	Far floc
2	2	0.1	45.46	42.10
		0.2	35.18	35.86
		1	28.00	28.99
		5	36.79	35.20
		10	43.97	39.55
	10	0.1	94.45	52.40
		0.2	73.35	45.63
		1	58.96	38.90
		5	71.70	44.14
		10	82.57	48.54
5	2	0.1	39.18	54.50
		0.2	29.70	43.58
		1	23.37	34.97
		5	31.81	42.91
		10	38.57	49.64
	10	0.1	78.16	54.00
		0.2	60.87	43.57
		1	47.90	35.29
		5	59.13	43.26
		10	68.57	49.81
10	2	0.1	37.88	63.30
		0.2	28.62	49.31
		1	22.48	38.68
		5	30.80	48.21
		10	37.41	56.32
	10	0.1	74.00	58.84
		0.2	57.68	46.15
		1	45.53	36.39
		5	56.60	45.49
		10	65.80	53.07

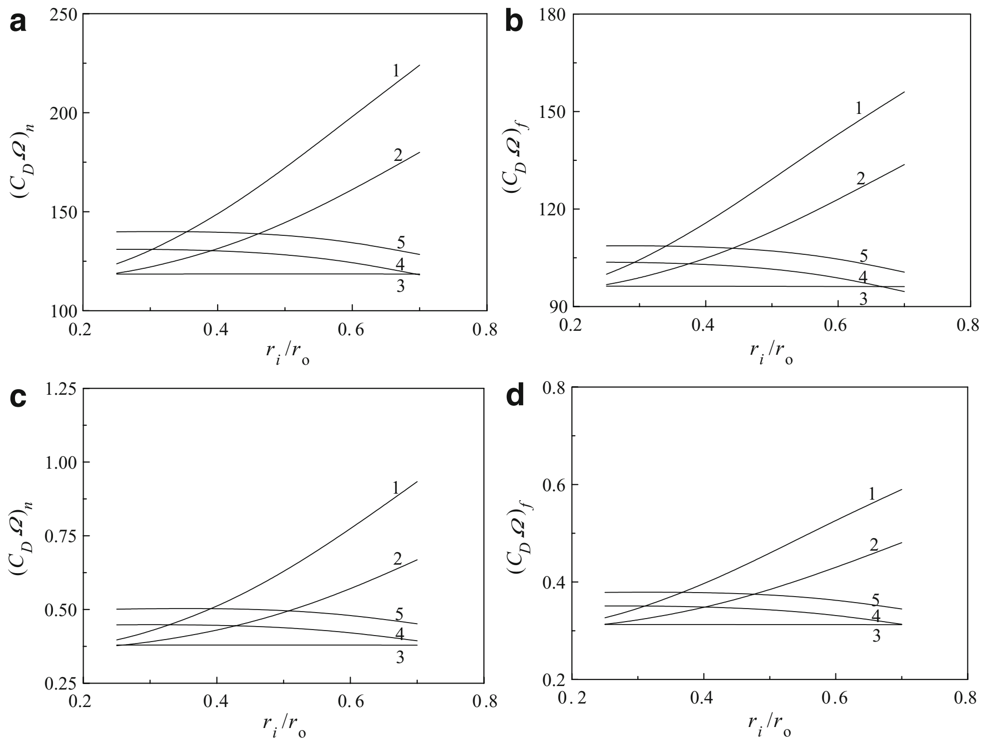

Fig. 8 Variations of $(C_D\Omega)_n$ (**a**, **c**) and $(C_D\Omega)_f$ (**b**, **d**), as a function of r_i/r_o for various combinations of k_o/k_i and *Re* at S/d=2, h_n/r_o=2, and $\overline{\beta} = 2$. *Curve 1* k_o/k_i=0.1, *2* k_o/k_i=0.2, *3* k_o/k_i=1, *4* k_o/k_i=5, *5* k_o/k_i=10. *Re*=0.1 in **a** and **b**, *Re*=40 in **c** and **d**

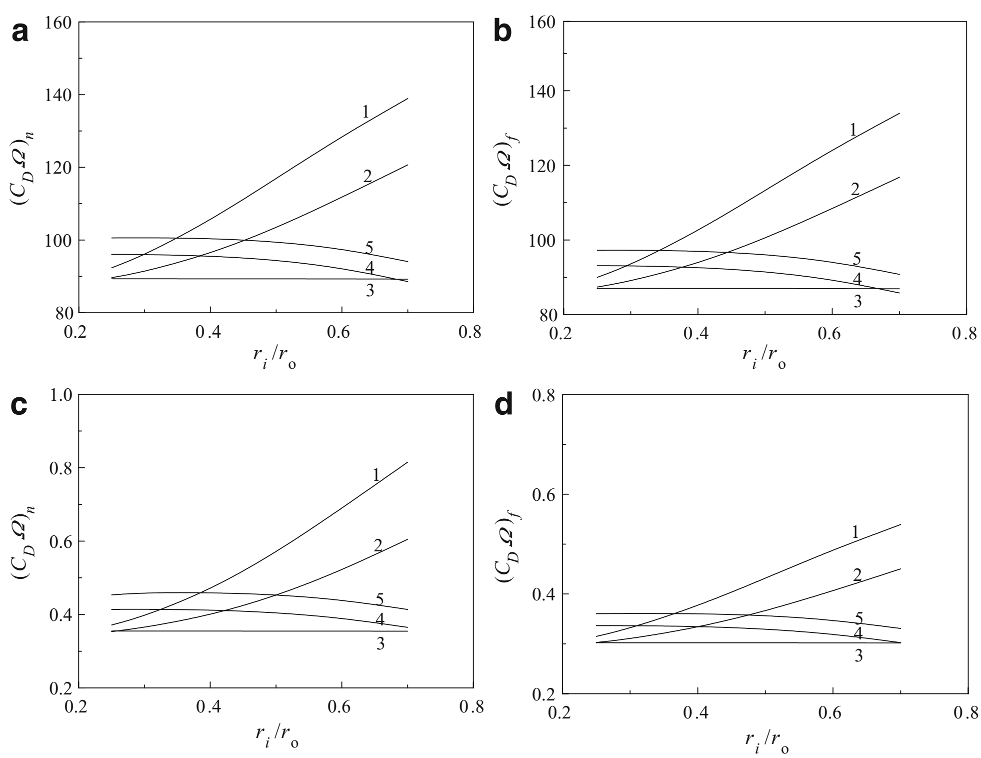

Fig. 9 Variations of $(C_D\Omega)_n$ (**a**, **c**) and $(C_D\Omega)_f$ (**b**, **d**), as a function of r_i/r_o for various combinations of k_o/k_i and *Re* at S/d=10, h_n/r_o=2, and $\overline{\beta} = 2$. *Curve 1* k_o/k_i=0.1, *2* k_o/k_i=0.2, *3* k_o/k_i=1, *4* k_o/k_i=5, *5* k_o/k_i=10. *Re*=0.1 in **a** and **b**, *Re*=40 in **c** and **d**

Table 3 Variation of $[(C_D\Omega(r_i/r_o = 0.7) - C_D\Omega(r_i/r_o = 0.25))/C_D\Omega(r_i/r_o = 0.25)] \times 100\%$ at various combinations of *Re*, (h_n/r_o), and (k_o/k_i) at S/d=2 and $\overline{\beta} = 2$

Re	h_n/r_o	k_o/k_i	$\frac{C_D\Omega(r_i/r_o=0.7)-C_D\Omega(r_i/r_o=0.25)}{C_D\Omega(r_i/r_o=0.25)} \times 100\%$	
			Near floc	Far floc
0.1	2	0.1	81.14	56.13
		0.2	51.37	38.19
		1	−0.01	−0.13
		5	−9.85	−8.72
		10	−8.17	−7.45
	10	0.1	50.42	48.83
		0.2	34.62	33.68
		1	−0.11	−0.11
		5	−7.79	−7.84
		10	−6.47	−6.62
40	2	0.1	135.24	80.64
		0.2	77.16	53.44
		1	−0.08	−0.14
		5	−12.12	−10.68
		10	−9.87	−8.95
	10	0.1	119.47	71.21
		0.2	70.99	48.76
		1	−0.18	−0.12
		5	−11.83	−10.09
		10	−8.75	−8.25

number, increasing ratio of center-to-center distance between two flocs to diameter of floc, and decreasing volume-average radius of a floc. Even if the volume-average permeability of a floc is fixed, the boundary effect still depends upon its structure. If the volume-average radius is fixed, the influence of the plate on the drag on a homogeneously structured floc is less significant than that of the corresponding nonhomogeneous floc, and the more nonhomogeneous the floc structure is, the more important that influence. In general, the smaller the Reynolds number, the more significant the boundary effect is. However, several exceptions are observed, and this phenomenon is pronounced if the volume-average permeability of a floc is small. (d) If Reynolds number exceeds approximate unity, the ln(corrected drag coefficient) against ln(Reynolds number) curve deviates positively from the Stokes' law. The percentage deviation for the near floc increases with decreasing ratio center-to-center distance between two flocs to diameter of floc or increasing ratio of distance between center of near floc and plate to radius of floc. The behavior of the percentage deviation of the far floc is not that regular as that of the near floc. If the ratio center-to-center distance between two flocs to diameter of floc is smaller than approximately 2, the general trend of the percentage deviation of the far floc is the same as that of the near floc, both declines with increasing significance of the boundary effect. However, if the ratio center-to-center distance between two flocs to diameter of floc exceeds approximately 10, although the percentage deviation of the near floc declines with the decreasing ratio of distance between center of near floc and plate to radius of floc, the percentage deviation of the far floc has the reverse trend. If the ratio distance between center of near floc and plate to radius of floc exceeds approximately 10, the percentage deviation of the far floc is smaller than that of the near floc, but the reverse trend is observed if that ratio is small. For both flocs, the more nonhomogeneous their structure, the larger is the percentage deviation. (e) For the case where the volume-average permeability of a floc is fixed, if the inner layer of the floc is more permeable than its outer layer, the larger the ratio of radius of inner layer/radius of outer layer, the larger the corrected drag coefficient. That trend is reversed if the inner layer of the floc is less permeable than its outer layer. The more nonhomogeneous the floc structure and/or the larger the Reynolds number, the more significant the influence of the ratio radius of inner layer to radius of outer layer on the corrected drag coefficient of both flocs. (f) An empirical relation, which correlates the corrected drag coefficient with the key parameters of the problem considered, is proposed for the case where the volume-average permeability of a floc is 2 and the ratio radius of inner layer to radius of outer layer is 0.5.

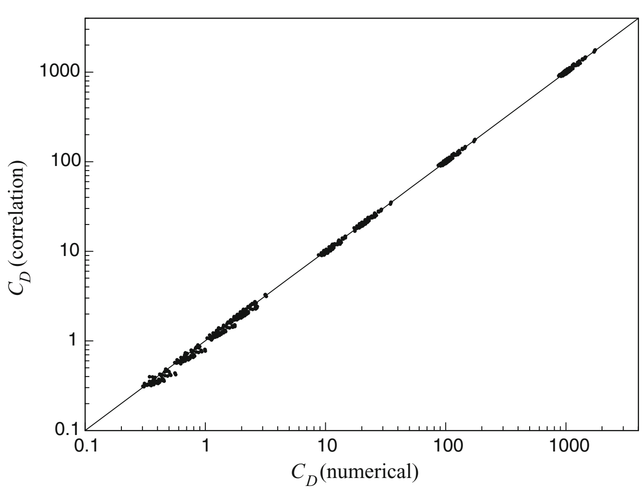

Fig. 10 Comparison of the calculated $C_D\Omega$ with that predicted by correlations

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